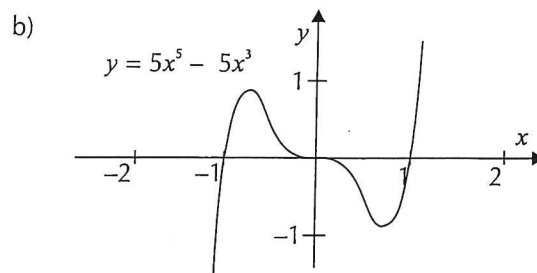
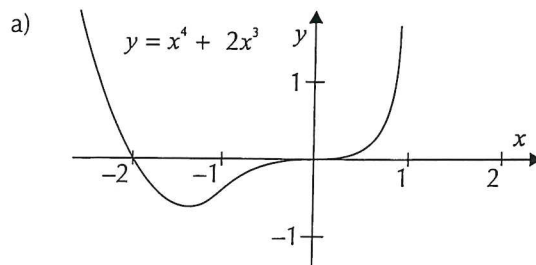


### Exercise 3.1

- Q1 Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  for each of these functions:
- a)  $y = x^3$       b)  $y = x^5$       c)  $y = x^4$       d)  $y = x$   
 e)  $y = \frac{1}{x}$       f)  $y = \sqrt{x}$       g)  $y = \frac{1}{x^2}$       h)  $y = x\sqrt{x}$
- Q2 Find  $f'(x)$  and  $f''(x)$  for each of these functions:
- a)  $f(x) = x(4x^2 - x)$       b)  $f(x) = (x^2 - 3)(x - 4)$   
 c)  $f(x) = \frac{4x^5 + 12x^3 - 40x}{4(x^2 + 5)}$       d)  $f(x) = 3\sqrt{x} + x\sqrt{x}$   
 e)  $f(x) = \frac{1}{x}(3x^4 - 2x^3)$       f)  $f(x) = \frac{x^2 - x\sqrt{x} + 7x}{\sqrt{x}}$
- Q3 Find the value of the second derivative at the given value for  $x$ .
- a)  $f(x) = x^3 - x^2$ ,  $x = 3$       b)  $y = x\sqrt{x} - \frac{1}{x}$ ,  $x = 4$   
 c)  $f(x) = x^2(x - 5)(x^2 + x)$ ,  $x = -1$       d)  $y = \frac{x^5 + 4x^4 - 12x^3}{x + 6}$ ,  $x = 5$   
 e)  $f(x) = \frac{9x^2 + 3x}{3\sqrt{x}}$ ,  $x = 1$       f)  $y = \left(\frac{1}{x^2} + \frac{1}{x}\right)(5 - x)$ ,  $x = -3$

### Exercise 3.2

- Q1 Without doing any calculations, say how many stationary points the graphs below have in the intervals shown.



- Q2 Find the  $x$ -coordinates of the stationary points of the curves with the following equations:
- a)  $y = x^2 + 3x + 2$       b)  $y = (3 - x)(4 + 2x)$
- Q3 Find the coordinates of the stationary points of the curves with the following equations:
- a)  $y = 2x^2 - 5x + 2$       b)  $y = -x^2 + 3x - 4$   
 c)  $y = 7 - 6x - 3x^2$       d)  $y = (x - 1)(2x + 3)$
- Q4 Find the coordinates of the stationary points of the curves with the following equations:
- a)  $y = x^3 - 3x + 2$       b)  $y = 4x^3 + 5$

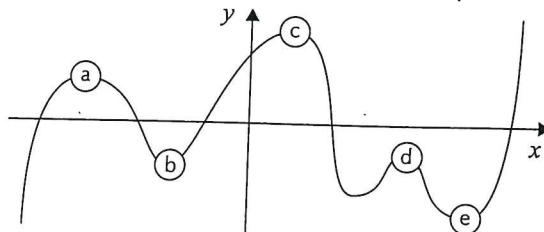
**Q5 Hint:** If there are no stationary points, there are no values of  $x$  for which  $f'(x) = 0$ .

- Q5 Show that the graph of the function given by  $f(x) = x^5 + 3x + 2$  has no stationary points.
- Q6 a) Differentiate  $y = x^3 - 7x^2 - 5x + 2$ .  
b) Hence find the coordinates of the stationary points of the curve with equation  $y = x^3 - 7x^2 - 5x + 2$ .
- Q7 A graph is given by the function  $f(x) = x^3 + kx$ , where  $k$  is a constant. Given that the graph has no stationary points, find the range of possible values for  $k$ .



### Exercise 3.3

- Q1 The diagram below shows a sketch of the graph of  $y = f(x)$ . For each turning point, say whether  $f''(x)$  would be positive or negative.



- Q2 For each of the following, find the second derivative of the function and say whether the given point is a maximum or a minimum:
- $y = x^3 - 12x + 4$  has a stationary point at  $(2, -12)$ .
  - $y = 2x^4 - 16x^3 + 900$  has a stationary point at  $(6, 36)$ .
  - $y = 4x^5 + 15x^4 - 250$  has a stationary point at  $(-3, -7)$ .
  - $y = x^5 - 5x^4 + 5x^2 - 40x + 400$  has a stationary point at  $(4, 64)$ .
- Q3 A function  $y = f(x)$  is such that  $f(1) = 3$ ,  $f'(1) = 0$  and  $f''(1) = 7$ .
- Give the coordinates of one of the turning points of  $f(x)$ .
  - Determine the nature of this turning point, explaining your answer.
- Q4 Find the stationary points on the graphs of the following functions and say whether they're maximum or minimum turning points:
- $y = 5 - x^2$
  - $y = 2x^3 - 6x + 2$
  - $y = x^3 - 3x^2 - 24x + 15$
  - $y = x^4 + 4x^3 + 4x^2 - 10$
- Q5 Find the stationary points on the graphs of the following functions and say whether they're maximum or minimum turning points:
- $f(x) = 8x^3 + 16x^2 + 8x + 1$
  - $f(x) = \frac{27}{x^3} + x$
- Q6 a) Given that  $f(x) = x^3 - 3x^2 + 4$ , find  $f'(x)$  and  $f''(x)$ .  
b) Hence find the coordinates of any stationary points on the graph  $f(x)$  and say whether they're maximum or minimum turning points.
- Q7 A function is given by  $y = x^2 + \frac{2000}{x}$ .
- Find the value of  $x$  at which  $y$  is stationary.
  - Is this a minimum or maximum point?
- Q8 The curve given by  $f(x) = x^3 + ax^2 + bx + c$  has a stationary point with coordinates  $(3, 10)$ . If  $f''(x) = 0$  at  $(3, 10)$ , find  $a$ ,  $b$  and  $c$ .
- Q9 a) Given that a curve with the equation  $y = x^4 + kx^3 + x^2 + 17$  has only one stationary point, show that  $k^2 < \frac{32}{9}$ .  
b) Find the coordinates of the stationary point and say whether it's a maximum or a minimum point.



### 3. Using Differentiation

#### Exercise 3.1 — Finding second order derivatives

Q1 a)  $\frac{dy}{dx} = 3x^2$  and  $\frac{d^2y}{dx^2} = 6x$ .

b)  $\frac{dy}{dx} = 5x^4$  and  $\frac{d^2y}{dx^2} = 20x^3$ .

c)  $\frac{dy}{dx} = 4x^3$  and  $\frac{d^2y}{dx^2} = 12x^2$ .

d)  $\frac{dy}{dx} = 1$  and  $\frac{d^2y}{dx^2} = 0$ .

e)  $y = x^{-1}$ , so  $\frac{dy}{dx} = -x^{-2} = -\frac{1}{x^2}$  and  $\frac{d^2y}{dx^2} = 2x^{-3} = \frac{2}{x^3}$ .

f)  $y = x^{\frac{1}{2}}$ , so  $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$   
and  $\frac{d^2y}{dx^2} = -\frac{1}{4}x^{-\frac{3}{2}} = -\frac{1}{4(\sqrt{x})^3}$ .

g)  $y = x^{-2}$ , so  $\frac{dy}{dx} = -2x^{-3} = -\frac{2}{x^3}$  and  $\frac{d^2y}{dx^2} = 6x^{-4} = \frac{6}{x^4}$ .

h)  $y = x\sqrt{x} = x^{\frac{3}{2}}$ , so  $\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$  and  $\frac{d^2y}{dx^2} = \frac{3}{4}x^{-\frac{1}{2}} = \frac{3}{4\sqrt{x}}$ .

Q2 a)  $f(x) = x(4x^2 - x) = 4x^3 - x^2$

$f'(x) = 12x^2 - 2x$

$f''(x) = 24x - 2$

b)  $f(x) = (x^2 - 3)(x - 4) = x^3 - 4x^2 - 3x + 12$

$f'(x) = 3x^2 - 8x - 3$

$f''(x) = 6x - 8$

c)  $f(x) = \frac{4x^5 + 12x^3 - 40x}{4(x^2 + 5)} = \frac{4x(x^4 + 3x^2 - 10)}{4(x^2 + 5)}$

$= \frac{4x(x^2 + 5)(x^2 - 2)}{4(x^2 + 5)} = x(x^2 - 2) = x^3 - 2x$

$f'(x) = 3x^2 - 2$

$f''(x) = 6x$

d)  $f(x) = 3x^{\frac{1}{2}} + xx^{\frac{1}{2}} = 3x^{\frac{1}{2}} + x^{\frac{3}{2}}$

$f'(x) = \frac{3}{2}x^{-\frac{1}{2}} + \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2\sqrt{x}} + \frac{3}{2}\sqrt{x}$

$f''(x) = \frac{3}{2}(-\frac{1}{2}x^{-\frac{3}{2}}) + \frac{3}{2}(\frac{1}{2}x^{-\frac{1}{2}}) = -\frac{3}{4}x^{-\frac{3}{2}} + \frac{3}{4}x^{-\frac{1}{2}}$   
 $= -\frac{3}{4(\sqrt{x})^3} + \frac{3}{4\sqrt{x}} = \left(-\frac{3}{4x\sqrt{x}} + \frac{3}{4\sqrt{x}}\right)$

e)  $f(x) = \frac{1}{x}(3x^4 - 2x^3) = 3x^3 - 2x^2$

$f'(x) = 9x^2 - 4x$

$f''(x) = 18x - 4$

f)  $f(x) = \frac{x^2 - xx^{\frac{1}{2}} + 7x}{x^{\frac{1}{2}}} = x^{\frac{3}{2}} - x^{\frac{1}{2}} + 7xx^{-\frac{1}{2}}$

$= x^{\frac{3}{2}} - x^{\frac{1}{2}} + 7x^{\frac{1}{2}}$

$f'(x) = \frac{3}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}} + 7(\frac{1}{2}x^{-\frac{1}{2}}) = \frac{3}{2}\sqrt{x} - \frac{1}{2\sqrt{x}} + \frac{7}{2\sqrt{x}}$

$f''(x) = \frac{3}{2}(\frac{1}{2}x^{-\frac{1}{2}}) + \frac{1}{4}x^{-\frac{3}{2}} + \frac{7}{2}(-\frac{1}{2}x^{-\frac{3}{2}}) = \frac{3}{4\sqrt{x}} - \frac{7}{4(\sqrt{x})^3}$

Q3 a)  $f'(x) = 3x^2 - 2x$ , so  $f''(x) = 6x - 2$ .

$f''(3) = 16$ .

b)  $y = xx^{\frac{1}{2}} - x^{-1} = x^{\frac{3}{2}} - x^{-1}$  so  $\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} + x^{-2}$

so  $\frac{d^2y}{dx^2} = \frac{3}{2}(\frac{1}{2}x^{-\frac{1}{2}}) - 2x^{-3} = \frac{3}{4\sqrt{x}} - \frac{2}{x^3}$

so at  $x = 4$ ,  $\frac{d^2y}{dx^2} = \frac{11}{32}$ .

c)  $f(x) = x^2(x^3 - 4x^2 - 5x) = x^5 - 4x^4 - 5x^3$

so  $f'(x) = 5x^4 - 16x^3 - 15x^2$

and  $f''(x) = 20x^3 - 48x^2 - 30x$ .

$f''(-1) = -38$ .

d)  $y = \frac{x^3(x+6)(x-2)}{(x+6)} = x^3(x-2) = x^4 - 2x^3$

so  $\frac{dy}{dx} = 4x^3 - 6x^2$ ,  $\frac{d^2y}{dx^2} = 12x^2 - 12x$ .

At  $x = 5$ ,  $\frac{d^2y}{dx^2} = 240$ .

e)  $f(x) = \frac{9x^2 + 3x}{3\sqrt{x}} = 3x^{\frac{3}{2}} + x^{\frac{1}{2}}$  so

$f'(x) = 3(\frac{3}{2}x^{\frac{1}{2}}) + \frac{1}{2}x^{-\frac{1}{2}} = \frac{9}{2}\sqrt{x} + \frac{1}{2\sqrt{x}}$  and so

$f''(x) = \frac{9}{2}(\frac{1}{2}x^{-\frac{1}{2}}) + \frac{1}{2}(-\frac{1}{2}x^{-\frac{3}{2}}) = \frac{9}{4\sqrt{x}} - \frac{1}{4(\sqrt{x})^3}$

$f''(1) = 2$ .

f)  $y = (x^2 + x^{-1})(5 - x)$

$= 5x^2 - x^2x + 5x^{-1} - xx^{-1}$

$= 5x^2 - x^3 + 5x^{-1} - 1 = 5x^2 + 4x^{-1} - 1$

$\frac{dy}{dx} = 5(-2x^3) + 4(-x^{-2}) = -10x^3 - 4x^{-2}$

so  $\frac{d^2y}{dx^2} = 30x^4 + 8x^{-3} = \frac{30}{x^4} + \frac{8}{x^3}$ .

At  $x = -3$ ,  $\frac{d^2y}{dx^2} = \frac{2}{27}$ .

#### Exercise 3.2 — Stationary points

Q1 a) The graph has 2 stationary points — a minimum and a point of inflection.

b) The graph has 3 stationary points — a maximum, a minimum and a point of inflection.

Q2 a)  $\frac{dy}{dx} = 2x + 3$ . When  $\frac{dy}{dx} = 0$ ,  $2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$

b)  $y = (3 - x)(4 + 2x) = 12 + 2x - 2x^2$

$\frac{dy}{dx} = 2 - 4x$ . When  $\frac{dy}{dx} = 0$ ,  $2 - 4x = 0 \Rightarrow x = \frac{1}{2}$

Q3 a)  $\frac{dy}{dx} = 4x - 5$ . When  $\frac{dy}{dx} = 0$ ,  $4x - 5 = 0 \Rightarrow x = \frac{5}{4}$

When  $x = \frac{5}{4}$ ,  $y = 2(\frac{5}{4})^2 - 5(\frac{5}{4}) + 2 = -\frac{9}{8}$   
So the coordinates are  $(\frac{5}{4}, -\frac{9}{8})$ .

b)  $\frac{dy}{dx} = -2x + 3$ . When  $\frac{dy}{dx} = 0$ ,  $-2x + 3 = 0$

$\Rightarrow x = \frac{3}{2}$ . When  $x = \frac{3}{2}$ ,  $y = -(\frac{3}{2})^2 + 3(\frac{3}{2}) - 4 = -\frac{7}{4}$

So the coordinates are  $(\frac{3}{2}, -\frac{7}{4})$ .

c)  $\frac{dy}{dx} = -6 - 6x$ . When  $\frac{dy}{dx} = 0$ ,  $-6 - 6x = 0 \Rightarrow x = -1$

When  $x = -1$ ,  $y = 7 - 6(-1) - 3(-1)^2 = 10$ .

So the coordinates are  $(-1, 10)$ .

d)  $y = (x - 1)(2x + 3) = 2x^2 + x - 3$

$\frac{dy}{dx} = 4x + 1$ . When  $\frac{dy}{dx} = 0$ ,  $4x + 1 = 0 \Rightarrow x = -\frac{1}{4}$

When  $x = -\frac{1}{4}$ ,  $y = (-\frac{1}{4} - 1)(2(-\frac{1}{4}) + 3) = -\frac{25}{8}$ .

So the coordinates are  $(-\frac{1}{4}, -\frac{25}{8})$ .

Q4 a)  $\frac{dy}{dx} = 3x^2 - 3$ . When  $\frac{dy}{dx} = 0$ ,  $3x^2 - 3 = 0 \Rightarrow x = \pm 1$

When  $x = 1$ ,  $y = 1^3 - 3(1) + 2 = 0$ .

When  $x = -1$ ,  $y = (-1)^3 - 3(-1) + 2 = 4$ .

So the coordinates are  $(1, 0)$  and  $(-1, 4)$ .

b)  $\frac{dy}{dx} = 12x^2$ . When  $\frac{dy}{dx} = 0$ ,  $12x^2 = 0 \Rightarrow x = 0$

When  $x = 0$ ,  $y = 4(0)^3 + 5 = 5$ .

So the coordinates are  $(0, 5)$ .

Q5  $f'(x) = 5x^4 + 3$ . When  $f'(x) = 0$ ,  $5x^4 + 3 = 0 \Rightarrow x^4 = -\frac{3}{5}$

Finding a solution would involve finding the fourth root of a negative number. But  $x^4 = (x^2)^2$ , so  $x^4$  is always positive and so there are no stationary points.

Q6 a)  $\frac{dy}{dx} = 3x^2 - 14x - 5$

b) When  $\frac{dy}{dx} = 0$ ,  $3x^2 - 14x - 5 = 0$

$\Rightarrow (3x + 1)(x - 5) = 0$ , so  $x = -\frac{1}{3}$  and  $x = 5$ .

When  $x = -\frac{1}{3}$ ,  $y = (-\frac{1}{3})^3 - 7(-\frac{1}{3})^2 - 5(-\frac{1}{3}) + 2 = \frac{7}{27}$

When  $x = 5$ ,  $y = 5^3 - 7(5)^2 - 5(5) + 2 = -73$ .

So the coordinates are  $(-\frac{1}{3}, \frac{7}{27})$  and  $(5, -73)$ .

Q7 For stationary points to occur,  $f'(x)$  must equal zero, so

$f'(x) = 3x^2 + k = 0 \Rightarrow -\frac{k}{3} = x^2$ . For this equation to

have a solution,  $k$  can't be positive (or it would be

taking the square root of a negative number), so  $k \leq 0$

Therefore, if the graph has no stationary points,  $k > 0$



### Exercise 3.3 — Maximum and minimum points

Q1 a) negative b) positive c) negative

d) negative e) positive

Q2 a)  $\frac{dy}{dx} = 3x^2 - 12$   $\frac{d^2y}{dx^2} = 6x$   
At (2, -12),  $\frac{d^2y}{dx^2} = 6 \times 2 = 12 > 0$ ,  
so (2, -12) is a minimum.

b)  $\frac{dy}{dx} = 8x^3 - 48x^2$   $\frac{d^2y}{dx^2} = 24x^2 - 96x$   
At (6, 36),  $\frac{d^2y}{dx^2} = 24 \times 6^2 - 96 \times 6 = 288 > 0$ ,  
so (6, 36) is a minimum.

c)  $\frac{dy}{dx} = 20x^4 + 60x^3$   $\frac{d^2y}{dx^2} = 80x^3 + 180x^2$   
At (-3, -7),  $\frac{d^2y}{dx^2} = 80 \times (-3)^3 + 180 \times (-3)^2$   
 $= -540 < 0$ , so (-3, -7) is a maximum.

d)  $\frac{dy}{dx} = 5x^4 - 20x^3 + 10x - 40$   
 $\frac{d^2y}{dx^2} = 20x^3 - 60x^2 + 10$   
At (4, 64),  $\frac{d^2y}{dx^2} = 20 \times 4^3 - 60 \times 4^2 + 10 = 330 > 0$ ,  
so (4, 64) is a minimum.

Q3 a) (1, 3)

All the clues are in the question — the derivative when  $x = 1$  is zero so you know it's a stationary point, and the  $y$ -value when  $x = 1$  is 3.

b) The second derivative at  $x = 1$  is positive, so it's a minimum.

Q4 a)  $\frac{dy}{dx} = -2x$ . When  $\frac{dy}{dx} = 0$ ,  $x = 0$ . When  $x = 0$ ,  $y = 5 - 0 = 5$ . So the coordinates are (0, 5).  
 $\frac{d^2y}{dx^2} = -2$ , so it's a maximum turning point.

b)  $\frac{dy}{dx} = 6x^2 - 6$ . When  $\frac{dy}{dx} = 0$ ,  $6x^2 = 6 \Rightarrow x = \pm 1$   
When  $x = 1$ ,  $y = 2 - 6 + 2 = -2$ . When  $x = -1$ ,  $y = -2 + 6 + 2 = 6$ . So the coordinates are (1, -2) and (-1, 6).  $\frac{d^2y}{dx^2} = 12x$ .

At (1, -2),  $\frac{d^2y}{dx^2} = 12$ , so it's a minimum.

At (-1, 6),  $\frac{d^2y}{dx^2} = -12$  so it's a maximum.

c)  $\frac{dy}{dx} = 3x^2 - 6x - 24$ . When  $\frac{dy}{dx} = 0$ ,  $x^2 - 2x - 8 = 0$   
 $\Rightarrow (x - 4)(x + 2) = 0 \Rightarrow x = 4$  and  $-2$ .  
When  $x = 4$ ,  $y = 64 - 48 - 96 + 15 = -65$ .  
When  $x = -2$ ,  $y = -8 - 12 + 48 + 15 = 43$ .  
So the coordinates are (4, -65) and (-2, 43).  
 $\frac{d^2y}{dx^2} = 6x - 6$ . At (4, -65),  $\frac{d^2y}{dx^2} = 24 - 6 = 18$ ,  
so it's a minimum.

At (-2, 43),  $\frac{d^2y}{dx^2} = -12 - 6 = -18$ , so it's a maximum.

d)  $\frac{dy}{dx} = 4x^3 + 12x^2 + 8x$ .

When  $\frac{dy}{dx} = 0$ ,  $x^3 + 3x^2 + 2x = 0$

$\Rightarrow x(x + 2)(x + 1) = 0$ , so  $x = 0$ ,  $-1$  and  $-2$ .

When  $x = 0$ ,  $y = 0 + 0 + 0 - 10 = -10$ .

When  $x = -1$ ,  $y = 1 - 4 + 4 - 10 = -9$ .

When  $x = -2$ ,  $y = 16 - 32 + 16 - 10 = -10$ .

So the stationary points are (0, -10), (-1, -9) and (-2, -10).

$\frac{d^2y}{dx^2} = 12x^2 + 24x + 8$ . At (0, -10),

$\frac{d^2y}{dx^2} = 0 + 0 + 8 = 8$ , so it's a minimum.

At (-1, -9),  $\frac{d^2y}{dx^2} = 12 - 24 + 8 = -4$ ,  
so it's a maximum.

At (-2, -10),  $\frac{d^2y}{dx^2} = 48 - 48 + 8 = 8$ ,  
so it's a minimum.

Q5 a)  $f'(x) = 24x^2 + 32x + 8$ .

When  $f'(x) = 0$ ,  $3x^2 + 4x + 1 = 0$

$\Rightarrow (3x + 1)(x + 1) = 0$ , so  $x = -1$  and  $-\frac{1}{3}$ .

When  $x = -1$ ,  $f(x) = -8 + 16 - 8 + 1 = 1$ .

When  $x = -\frac{1}{3}$ ,  $f(x) = -\frac{8}{27} + \frac{16}{9} - \frac{8}{3} + 1 = -\frac{5}{27}$ .

So the coordinates are (-1, 1) and  $(-\frac{1}{3}, -\frac{5}{27})$ .

$f''(x) = 48x + 32$ . At (-1, 1)  $f'(x) = -48 + 32 = -16$ ,  
so it's a maximum.

At  $(-\frac{1}{3}, -\frac{5}{27})$ ,  $f''(x) = -\frac{48}{3} + 32 = 16$ ,

so it's a minimum.

b)  $f(x) = \frac{27}{x^3} + x = 27x^{-3} + x \Rightarrow f'(x) = -81x^{-4} + 1$ .

When  $f'(x) = 0$ ,  $x^4 = 81 \Rightarrow x = \pm 3$ .

When  $x = 3$ ,  $f(x) = \frac{27}{27} + 3 = 4$ .

When  $x = -3$ ,  $f(x) = -\frac{27}{27} - 3 = -4$ .

So the coordinates are (3, 4) and (-3, -4).

$f''(x) = 324x^{-5}$ . At (3, 4)  $f''(x) = \frac{4}{3}$ ,

so it's a minimum.

At (-3, -4)  $f''(x) = -\frac{4}{3}$ , so it's a maximum.

Q6 a)  $f'(x) = 3x^2 - 6x$ .  $f''(x) = 6x - 6$ .

b) When  $f'(x) = 0$ ,  $3x^2 - 6x = 0 \Rightarrow x(x - 2) = 0$ ,

so  $x = 0$  and  $x = 2$ . When  $x = 0$ ,  $f(x) = 0 - 0 + 4 = 4$ .

When  $x = 2$ ,  $f(x) = 8 - 12 + 4 = 0$ .

So the coordinates are (0, 4) and (2, 0).

At (0, 4)  $f''(x) = 0 - 6 = -6$ , so it's a maximum.

At (2, 0)  $f''(x) = 12 - 6 = 6$ , so it's a minimum.

Q7 a)  $y = x^2 + \frac{2000}{x} = x^2 + 2000x^{-1} \Rightarrow \frac{dy}{dx} = 2x - \frac{2000}{x^2}$

When  $\frac{dy}{dx} = 0$ ,  $2x = \frac{2000}{x^2} \Rightarrow x^3 = 1000 \Rightarrow x = 10$

b)  $\frac{d^2y}{dx^2} = 2 + \frac{4000}{x^3}$ . When  $x = 10$ ,

$\frac{d^2y}{dx^2} = 2 + 4 = 6$ , so it's a minimum.

Q8  $f(x) = x^3 + ax^2 + bx + c \Rightarrow f'(x) = 3x^2 + 2ax + b$ .

$\Rightarrow f''(x) = 6x + 2a$ . At the point (3, 10):

$10 = 3^3 + a(3^2) + b(3) + c \Rightarrow 10 = 27 + 9a + 3b + c$

As (3, 10) is a stationary point,  $0 = 3(3^2) + 2a(3) + b$

$\Rightarrow 0 = 27 + 6a + b$ . We know that  $f''(3) = 0$ ,

so  $0 = 6(3) + 2a \Rightarrow 0 = 18 + 2a \Rightarrow a = -9$ .

Then  $0 = 27 + 6a + b = 27 + 6(-9) + b \Rightarrow b = 27$

And  $10 = 27 + 9a + 3b + c = 27 + 9(-9) + 3(27) + c$

$\Rightarrow c = -17$ . So  $f(x) = x^3 - 9x^2 + 27x - 17$ .

Q9 a)  $\frac{dy}{dx} = 4x^3 + 3kx^2 + 2x$ .

Stationary points occur when  $\frac{dy}{dx} = 0$ ,

so  $4x^3 + 3kx^2 + 2x = 0 \Rightarrow x(4x^2 + 3kx + 2) = 0$

so  $x = 0$  or  $4x^2 + 3kx + 2 = 0$ .

As you know the only stationary point occurs at  $x = 0$ , the part in brackets can't have any solutions.

This gives you information about the discriminant of the quadratic equation:

$b^2 - 4ac < 0 \Rightarrow 9k^2 < 32 \Rightarrow k^2 < \frac{32}{9}$ .

b) When  $x = 0$ ,  $y = 0 + 0 + 0 + 17 = 17$ , so the coordinates are (0, 17).

$\frac{d^2y}{dx^2} = 12x^2 + 6kx + 2$ . When  $x = 0$ ,  $\frac{d^2y}{dx^2} = 2$ ,

so it's a minimum.